Sample Questions for Algorithms Ph.D Qualifier

You have 90 minutes to complete this exam.

Please use the test paper for your work.

Do not put your name anywhere on this paper.

Enter your exam code number here: __________

Problem 1
Design a divide-and-conquer algorithm to multiply two n-bit integers. The algorithm shall run in $O(n^{log_2 3})$ time. Describe your algorithm and provide run-time analysis.

Problem 2
To implement a k-bit binary counter, we use array $A[k]$ to store the bits. The highest-order bit is in $A[k-1]$ and the lowest-order bit is in $A[0]$, so the value x stored in the binary counter is: $x = \sum_{i=0}^{k-1} A[i] \cdot 2^i$. Each call to procedure INCREMENT increases x by one. The following example shows a 4-bit binary counter as its value goes from 0 to 5 by a sequence of 5 INCREMENT operations.

0000 INCREMENT $\rightarrow$
0001 INCREMENT $\rightarrow$
0010 INCREMENT $\rightarrow$
0011 INCREMENT $\rightarrow$
0100 INCREMENT $\rightarrow$
0101

Suppose to flip a bit costs 1. For a sequence of n INCREMENT operations on an initially zero counter, what is the amortized cost per operation?

Problem 3
Give asymptotic tight bounds for $T(n)$ in the following recurrences.
Write down the final answers only. No partial credit will be given if the answer is wrong. Assume that $T(n)$ is constant for sufficiently small n.
3-1 $T(n) = 2T(n/2) + n/\lg n$

3-2 $T(n) = T(9n/10) + n$

3-3 $T(n) = 2T(n/2) + n \lg n$

3-4 $T(n) = 8T(n/2) + n \lg n$

3-5 $T(n) = T(n-1) + n$

Problem 4
Show that if $f(n) = \Theta(n^{log_b a} lg^k n)$, where $k \geq 0$, then the master recurrence has solution $T(n) = \Theta(n^{log_b a} lg^{k+1} n)$. For simplicity, confine your analysis to exact powers of b.

Problem 5
Which of the following problems are NP-complete? check all that apply.
_____**5-1** The Euler tour problem: An Euler tour of a connected, directed graph G is a cycle that traverses each edge of G exactly once. The Euler tour problem is to determine whether a graph has an Euler tour.

_____**5-2** DNF-SAT problem: given a boolean formula in disjunctive normal form(DNF), determine if it is satisfiable. DNF formula example: $\phi = (y1 \wedge y2 \wedge \neg y3) \vee (y1 \wedge \neg y2 \wedge x2)$

_____**5-3** Vertex cover problem: For a given graph $G = (V, E)$, a vertex cover is a subset $V' \subseteq V$ such that every edge has at least one end point in V'. The vertex cover problem is to determine if there exists a vertex cover of size K in a given graph G.

_____**5-4** The Hamiltonian Path problem: a Hamiltonian path in a graph G is a simple path that visits every vertex exactly once. The Hamiltonian Path problem is to determine whether there is a Hamiltonian path from u to v in G.

_____**5-5** 3CNF-SAT: given a boolean formula in 3CNF form, determine whether it is satisfiable. 3CNF formula example:$\phi = (y1 \vee y2 \vee \neg y3) \wedge (y1 \vee \neg y2 \vee x2)$