

# Quantification of Information Flow in a Smart Grid

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**Abstract**—The key to computer security is the notion of information flow. Information flow occurs either explicitly or implicitly in a system. In cyber-physical systems (CPSs), complicated interactions occur frequently between computational components and physical components. Thus, detecting and quantifying information flow in these systems is more difficult than it is in purely cyber systems. In CPSs, failures and attacks are either from the physical infrastructure, or from cyber part of data management and communication protocol, or a combination of both. As the physical infrastructure is inherently observable, aggregated physical observations can lead to unintended cyber information leakage. The computational portion of a CPS is driven by algorithms. Within algorithmic theory, the online problem considers input that arrives one by one and deals with extracting the algorithmic solution through an advice tape without knowing some parts of input. In this paper, a smart grid CPS is examined from an information flow perspective; physical values constitute an advice tape. As such, system confidentiality is violated through cyber to physical information flow. An approach is generalized to quantify the information flow in a CPS.

**Keywords**-Information Flow, Advice Tape, Online Problem, Quantification

## I. INTRODUCTION

Computer security has three principal categories: confidentiality, integrity, and availability. Confidentiality prohibits the unauthorized reading of information. Integrity prohibits the unauthorized modifying of information, and availability prohibits unauthorized withholding of information. Standard security mechanisms (e.g., access control, firewalls, encryption and antivirus software) prevent, to some extent, confidentiality violations. They do not, however, provide end-to-end security because indirect information transmission usually occurs in covert channels that are not easy to detect. Information flow analysis works as a compensation for standard mechanisms by controlling the information flow among different entities to ensure confidentiality. It specifies the restrictions on a system's input-output relation to rule out non-secure implementation. It does a better job than standard security mechanisms for identifying information leakages in covert channels [1]. Three information flow properties (non-interference, non-inference, and non-deducibility) have widely been discussed in the literature [1], [2].

However, it is more difficult to apply information flow analysis in a Cyber-Physical System (CPS) than in purely cyber system, in which a set of networked computational components automatically control and monitor distributed physical entities. In CPSs, failures and attacks are either from the physical infrastructure, or from the cyber part of data management and communication protocols, or a combination of both [3].

A typical example of a CPS examined in this paper, the smart grid, uses intelligent transmission and distribution networks to deliver electricity. Numerous vulnerabilities and challenges arise with the smart grid, as it involves many factors (i.e., the cyber part, the physical part, human behavior, and commercial interests). In one side, a cyber attack will lead to eavesdropping of private information. A cyber attack is also used to affect physical system such as Stuxnet. In another side, the physical attacks can influence the cyber by compromising the data by using a shunt to a meter [4].

This paper examines information flow in the Future Renewable Electric Energy Delivery and Management (FREEDM) system. Distributed Grid Intelligence (DGI) in the FREEDM system the cyber process performs intelligent, distributed computation and management. As an increased number of physical observations are obtained, potential confidentiality violations can be quantified for FREEDM. A method is presented here to analyze information leakage by applying the advice tape concept in the field of algorithms.

The remainder of this paper consists of three main sections. Section 2 introduces the background of FREEDM DGI with a particular focus on the algorithm used for power migration and balance. Both the online algorithm and advice complexity are given a brief description in Section 3. Section 4 presents an analysis of information flow quantification in FREEDM. Experiments and results are given in Section 5. Finally, the conclusion as well as work proposed for the future are presented in section 6.

## II. SYSTEM BACKGROUND

DGIs in FREEDM implement balancing power flow and optimal distribution of energy through their distributed load balancing algorithm [5]. Each node in FREEDM consists of

a Solid State Transformer (SST), a Distributed Renewable Energy Resource (DRER), a Distributed Energy Storage Device (DESD), and a LOAD (the consumption of power at the household). Each node computes the local SST's actual load based on the above information.

DGIs use the distributed load balancing algorithm in [6] to manage power in FREEDM. First, nodes in the system form a group with a group leader based on a leader election algorithm. In the load balancing algorithm, each node in FREEDM will receive a Normal value from the group leader. Then the node will be classified as Supply if its SST's actual load value is larger than the Normal value. It will be classified as Demand if its SST's actual load value is less than the Normal value. The nodes participating in the load balancing algorithm communicate their load changes with each other in order to migrate power from the Supply node to the Demand node. The nodes normalize their loads and achieve a relatively balanced load status after the power migrations are complete.

Each DGI maintains a Load Table (as presented in Table I), which stores information of its local grid organization and information from other nodes in the system.

Table I  
FREEDM LOAD TABLE: POWER MANAGEMENT

Net DRER: __	Net DESD: __
Net LOAD: __	SST Gateway: __
Normal: __	
<b>Node</b>	<b>State</b>
Node 1	Demand
Node 2	Supply
...	...
Node n	Normal

### III. ONLINE PROBLEMS AND ADVICE TAPE

Online problems are problems that know only a part of their input at any specific point during the runtime [7]. The respective algorithms are known as online algorithms. Unlike offline algorithms (in which the input is known to the algorithm at the beginning of the computation), the input is received piece-by-piece in online algorithms. After receiving one piece of the input, the algorithm must make a decision on the solution and cannot reverse this decision in a later phase. The algorithm for the online problem tries to generate a solution with zero knowledge about some missing instances in the input.

Competitive analysis is the standard method used to measure the quality of an online algorithm. The competitive ratio is defined as the quality of the solution by the online algorithm over the quality of an offline algorithm.

The advice tape is another method proposed to achieve an optimal competitive ratio for online problems [8]. It contains a number of bits information that are necessary and sufficient for the forthcoming inputs. In the model of advice tape presented in Figure 1, an oracle (which sees the

entire input and has unlimited computational power), would communicate passively with the online algorithm. The oracle will write bitwise information onto an advice tape. Then the advice tape will be read by the online algorithm in a sequential order before reading any input  $x_1, x_2, \dots, x_n$ .

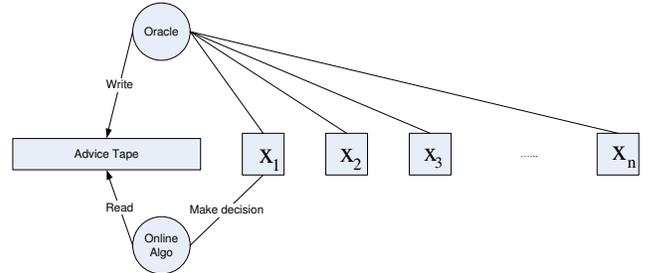


Figure 1. Advice Tape Model

The advice complexity for online problems describes how many bits of advice about the unknown parts of the input are necessary and sufficient to achieve a specific competitive ratio. It is obvious that a linear number of advice bits which indicate each input leads to an offline algorithm. While a large amount of advice is necessary to achieve an optimal solution, it is usually the case that few advice bits are sufficient to improve the competitive ratio [9].

The knapsack problem is a well-studied optimization problem. In the knapsack problem, a set of items with specified weights and profits are provided with a knapsack capacity. The goal is to choose a subset of items with maximum possible profits such that their total weights do not exceed the knapsack's capacity.

*Definition 3.1 (Online Knapsack Problem):* The input consists of a sequence of  $n$  items that are tuples of weights and profits,  $X = \{x_1, x_2, \dots, x_n\}$ ,  $x_i = (w_i, p_i)$ , where  $0 < w_i \leq 1$  and  $p_i > 0$  for  $i \in \{1, 2, \dots, n\}$ . A feasible solution is any subset  $X'$  of  $X$  such that  $\sum w_i \leq 1$ ; the goal is to maximize  $\sum p_i$  for  $i \in X'$ . The items are given an online fashion. For each item, an online algorithm  $A$  must specify whether or not this item is part of solution as soon as it is offered. In a simple online knapsack, each item's profit is smaller than 1 and equals its weight.

The advice complexity of the online knapsack problem has been thoroughly analyzed. An advice tape that contains the following three parts (as listed in Table II) will satisfy the competitive ratio  $(1 + \epsilon)$  to a simple online knapsack [10].

Table II  
ADVICE TAPE FOR SIMPLE ONLINE KNAPSACK

First bit	$j$	Index 1	...	Index $j$	$sum_{light}$
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- The first bit indicates whether or not there exists any item within the input of size  $> \delta$ .

- The heavy item is recorded with both an index and the total number of heavy items.
- The light items are recorded with lower bound summation.

#### IV. QUANTIFICATION ANALYSIS OF INFORMATION FLOW IN SMART GRID

The implicit information flow between the cyber part and the physical part, particularly the aggregated physical observation under the process of a cyber algorithm, is the focus of this paper. As shown in Figure 2, the observer would be an outsider  $A_n$  or be inside of the system as  $A_1$  or  $A_2$ . This section will analyze the information leakage based on the observer's position to the system and especially focus on the situation that observers would communicate with each other.

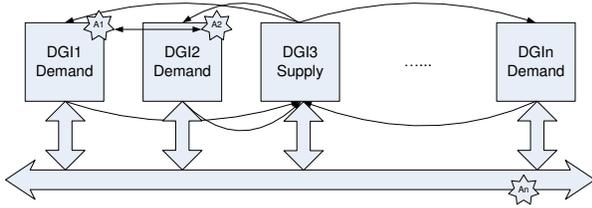


Figure 2. Observability Attacks of FREEDM

##### A. External Observer Obtaining One Reading without DGI

Power lines are physically visible and public; an observer could obtain a reading on one node. The information either is the message passing in the Load Balancing algorithm or the input power that is transmitted among the nodes. The system suffers from information leakage.

Suppose there are  $n$  demand values  $x_1, x_2, \dots, x_n$ , the number of possible supply value will be  $2^n$ :  $X_1, X_2, \dots, X_{2^n}$ . Assume the supply value is chosen under a normal distribution, the probability for each value would be  $\frac{1}{2^n}$ . The Shannon Entropy of the system is:

$$H(X) = \sum P(X) \log \frac{1}{P(X)} = \sum \frac{1}{2^n} \log 2^n = n \quad (1)$$

If only one value  $x_i$  is observed to the observer through the power line, the number of possible values for the supply value is  $2^{n-1}$ . Because  $X$  is under a normal distribution,  $H(X|x_i)$  can be obtained according to Shannon Entropy:

$$H(X|x_i) = \sum \frac{1}{2^{n-1}} \log 2^{n-1} = n - 1 \quad (2)$$

The information leakage is calculated as

$$I = H(X) - H(X|x_i) = n - (n - 1) = 1 \quad (3)$$

##### B. Multiple External Observers without DGI

More observers can obtain more readings on the shared power bus. If two values  $x_i$  and  $x_j$  are revealed to the observer, the number of possible values for the supply value is  $2^{n-2}$ . Assuming  $X$  is under a normal distribution,  $H(X|x_i, x_j)$  can be obtained according to Shannon's Entropy theory:

$$H(X|x_i, x_j) = \sum \frac{1}{2^{n-2}} \log 2^{n-2} = n - 2 \quad (4)$$

The information leakage is calculated as

$$I = H(X) - H(X|x_i, x_j) = n - (n - 2) = 2 \quad (5)$$

As the values are given to the observer one-by-one, the number of possible values for the supply value is changed from  $2^n, 2^{n-1}, 2^{n-2}$ , until all of the values are revealed. The information leakage is increased linearly as each value is revealed.

##### C. Internal Observer within DGI as Advice Tape

With messages and Load Table provided by DGI, the observer tries to deduce the total supply value as his observations increase. The attack model in this paper is similar to the advice tape model as shown in Figure 3: the system is the oracle which sees all the values and applies a knapsack algorithm on them; the observer is the online algorithm that tries to obtain a competitive ratio of the optimal solution. And the observer is seeing the messages and Load Table work as the advice tape which has been provided by the physical observations and cyber algorithm.

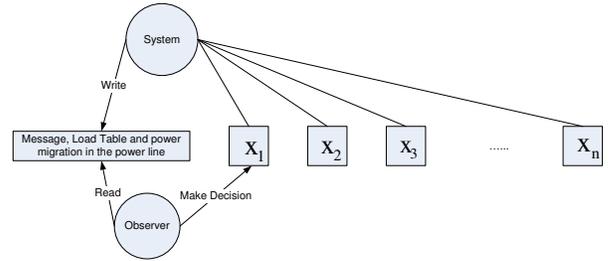


Figure 3. Advice Tape Model for DGI

*Lemma 4.1:* Within DGI, Shannon Entropy is not an appropriate measurement for the information leakage as it does not consider the ratio of revealed part over the secret. The observer will deduce the lower bound ratio of revealed physical observations over the secret.

*Proof:* In the previous two sections, Shannon Entropy is used to measure the information leakage as the observer is outside the DGI. If  $x_i$  is revealed as a solution among  $x_1, x_2, \dots, x_n$  through physical observation, Shannon entropy is calculated as

$$H(X|x_i) = \sum \frac{1}{2^{n-1}} \log 2^{n-1} = n - 1 \quad (6)$$

It means that a large observed value will lead to the same information uncertainty as a small observed value. However, it is not appropriate to measure the information uncertainty for the observers within DGI. Because the Load Table will display all values from demand nodes, the supply value will be bounded by the sum of all demand values. Thus, each individual physical observation in the power line will reveal a ratio to the bounded supply value. A large observed value will have different ratio to the supply value with a small observed value. ■

A function should be defined to measure the uncertainty of the information by considering both the number of revealed items and the ratio of the revealed part over the secret. The function is defined as  $F$  metric. The uncertainty of the information is between 0 and 1. 0 means the observer knows nothing about the information while 1 means the observer knows all about the information. If there is no item revealed, the uncertainty of the information is defined as 0. If there is at least one item revealed, the uncertainty of the information would be calculated by the  $F$  metric in the following.

$$F = \frac{2 \times \frac{h}{t} \times \frac{V}{X}}{\frac{h}{t} + \frac{V}{X}} \quad (7)$$

$h$  is the number of revealed items;  $t$  is the number of items in the optimal solution;  $V$  is the value of revealed items;  $X$  is the value for the optimal solution. It is straightforward to obtain that  $F = 1$  when  $h = t$  and  $V = X$ .

*Theorem 4.1:* The 0-1 knapsack problem can be quantified information leakage by considering a threshold (decided by  $\delta$ ) to the constraint which differentiates the heavy and light items in the solution.

*Proof:* Recall that advice tape has a fixed  $\delta$  which will differentiate items in solution into two part: heavy item is larger than  $\delta$  and light item is smaller than  $\delta$ . Let  $f$  represent the flag for a heavy item and  $m$  represent the number of heavy items in the advice tape.

When the first bit  $f$  is revealed, Table III shows the boundary of  $X$ .

Table III  
FIRST BIT REVEALED INFORMATION

First Bit	$x_1$	$x_2$	...	$x_n$
0				$\frac{x_n}{X} \leq \delta \Rightarrow X \geq \frac{x_n}{\delta}$
1				$\frac{x_n}{X} > \delta \Rightarrow X < \frac{x_n}{\delta}$

If the first bit and the number of heavy items are revealed as 1 and  $m$ , respectively, the lower bound of  $X$  will be the sum of  $m$  smallest values in the Load Table, and the upper bound will be the maximum value over  $\delta$  :

$$x_1 + x_2 + \dots + x_m < X < \frac{x_n}{\delta} \quad (8)$$

As each heavy item is revealed, both the lower bound and the upper bound of  $X$  are updated correspondingly. If only

one item  $x_i$  is revealed in advice tape as a heavy item, then

$$x_1 + x_2 + \dots + x_{m-1} + x_i < X < \frac{x_i}{\delta} \quad (9)$$

If two items  $x_i$  and  $x_j$  are revealed as heavy items, then

$$x_1 + x_2 + \dots + x_{m-2} + x_i + x_j < X < \min\left(\frac{x_i}{\delta}, \frac{x_j}{\delta}\right) \quad (10)$$

If a light item  $x_k$  is shown next, then

$$\max((x_1 + \dots + x_{m-3} + x_i + x_j + x_k), \frac{x_k}{\delta}) < X < \min\left(\frac{x_i}{\delta}, \frac{x_j}{\delta}\right) \quad (11)$$

As more and more items are revealed, both the lower bound and the upper bound of  $X$  are updated, the more the certainty of the constraint in the solution has.

As the advice tape (with a fixed  $\delta$ ) is revealed bit by bit, the boundary of  $X$  is given in Table IV. ■

## V. EXPERIMENTS AND RESULTS

Within DGI, the observer can use the Load Table to calculate the average value among the demand nodes. A big observed power migration in the power line (larger than the average value) indicates there is a heavy item in the solution; while a small observed power migration in the power line (less than the average value) indicates there is a light item in the solution. A selection of a DGI in the Load Balance is an indication that the observed node is in the solution. The attacker can obtain multiple readings from its neighbors. As the attacker observes more and more items, the boundary of constraint will be calculated out by differentiating it as a heavy item or a light item based on a  $\delta$ . The following example presents some results of quantifying uncertainty information of supply value (the constraint) from FREEDM.

### A. Case 1: Six items as solutions to a knapsack problem

The Load Table in DGI shows demand values 30, 60, 75, 200, 45, 20, 90, 120, 39, 47, 54, 108, 112 in the system with total summation 1000. Suppose the optimal solution for supply value 422 is 45, 20, 90, 120, 39, 108.

For  $\delta = 0.2$ , the heavy items are 90, 120, 108 because the threshold is  $422 \times 0.2 = 84.4$ . The total numbers of different sequences of items in the solution are  $6! = 720$ . The Figure 4 shows the uncertainty family curves for six items showing up in different orders with a  $\delta = 0.2$ . The uncertainty portion is the shaded area.

For a  $\delta = 0.08$ , the heavy items are 45, 39, 90, 120, 108 because the threshold is  $422 \times 0.08 = 33.76$ . The Figure 5 shows the whole uncertainty family curves for six items showing up in different orders with a smaller  $\delta = 0.08$ . The uncertainty portion is the shaded area.

The similarity of Figure 4 and Figure 5 shows that for a specific knapsack problem different  $\delta$  won't make much change to the shape of the uncertainty portion as long as the heavy part contains the item that is close to the threshold.

Table IV  
INFORMATION LEAKAGE

Bit meaning	Number of Bits in Advice Tape	Boundary of $X$	F Uncertainty
$f$	1	$\frac{x_m}{\delta} > X$	$F > \frac{2 \times \frac{1}{t} \times \delta}{\frac{1}{t} + \delta}$
$f, m$	$< 1 + \lceil \log \frac{1}{\delta} \rceil$	$\frac{x_m}{\delta} > X > x_1 + x_2 + \dots + x_m$	$F > \frac{2 \times \frac{m}{t} \times m \times \delta}{\frac{1}{t} + m \times \delta}$
$f, m, h_1$	$< 1 + \lceil \log \frac{1}{\delta} \rceil + \lceil \log n \rceil$	$\frac{h_i}{\delta} > X > x_1 + x_2 + \dots + x_{m-1} + h_1$	$F > \frac{2 \times \frac{m}{t} \times \frac{h_1}{X} + (m-1) \times \delta}{\frac{m}{t} + \frac{h_1}{X} + (m-1) \times \delta}$
$f, m, h_1, h_2$	$< 1 + \lceil \log \frac{1}{\delta} \rceil + 2 \lceil \log n \rceil$	$\min(\frac{h_1}{\delta}, \frac{h_2}{\delta}) > X > x_1 + x_2 + \dots + x_{m-2} + h_1 + h_2$	$F > \frac{2 \times \frac{m}{t} \times \frac{h_1+h_2}{X} \times (m-2) \times \delta}{\frac{m}{t} + \frac{h_1+h_2}{X} + (m-2) \times \delta}$
...	...	...	...
$f, m, h_1, \dots, h_m$	$< 1 + \lceil \log \frac{1}{\delta} \rceil + \lceil \log \frac{1}{\delta} \rceil \lceil \log n \rceil$	$\min(\frac{h_1}{\delta}, \frac{h_2}{\delta}, \dots, \frac{h_m}{\delta}) > X > h_1 + h_2 + \dots + h_m$	$F > \frac{2 \times \frac{m}{t} \times \frac{h_1+h_2+\dots+h_m}{X}}{\frac{m}{t} + \frac{h_1+h_2+\dots+h_m}{X}}$

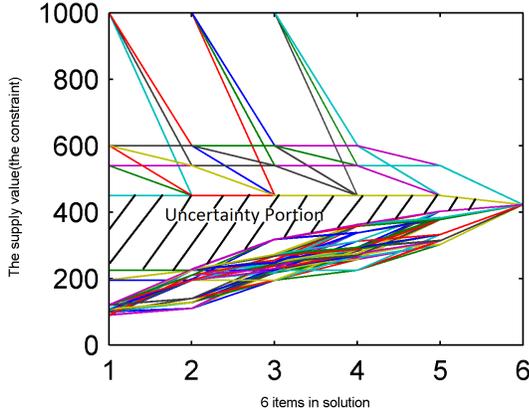


Figure 4. Uncertainty Portion with 6 items and  $\delta = 0.2$

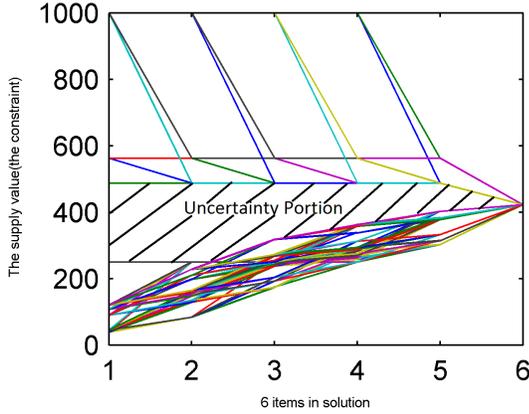


Figure 5. Uncertainty Portion with 6 items and  $\delta = 0.08$

### B. Case 2: Split the light items in Case 1

The knapsack problem has been kept the heavy items unchanged and split the light items to make bigger differences between the heavy part and the light part.

Suppose the problem is 30, 60, 75, 200, 21, 14, 10, 9, 90, 120, 10, 17, 22, 47, 54, 108, 112 and the optimal solution for constraint 422 is 21, 14, 9, 11, 90, 120, 10, 17, 22, 108.

For those 10 items in the solution, the heavy items are 90, 108, 120 and the possible sequences of those 10 items are 10!. Partial sequences have been picked to show the uncertainty curves for  $\delta = 0.2$  in Figure 6 and  $\delta = 0.08$  in Figure 7.

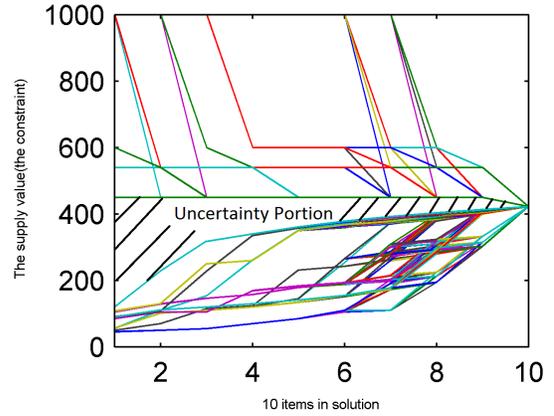


Figure 6. Uncertainty Portion with 10 items and  $\delta = 0.2$

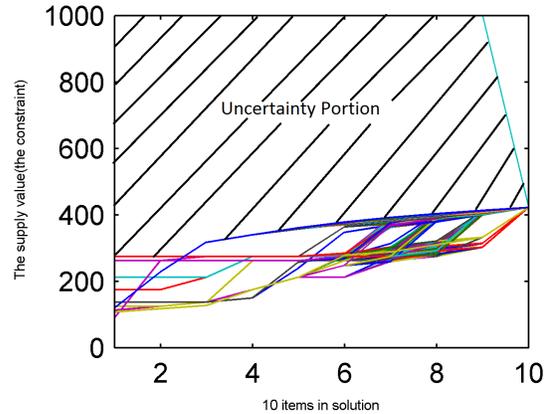


Figure 7. Uncertainty Portion with 10 items and  $\delta = 0.08$

In Figure 7, the uncertainty portion that is the shaded

area between the upper bounds and the lower bounds is much bigger than those of the previous. The upper bound of the constraint has not changed until the last item shows up. The difference between Figure 6 and Figure 7 shows that given the total sum of all items in the system is much larger than the individual item, heavy part that is far from the threshold will lead to more uncertainty of the constraint.

### C. Case 3: Split the heavy items in Case 1

If the heavy items have been split into smaller and similar values: 30, 60, 75, 66, 44, 62, 26, 45, 12, 8, 42, 48, 73, 47, 55, 53, 39, 49, 54, 43, 69, the optimal solution for constraint 422 is 45, 12, 8, 42, 48, 73, 47, 55, 53, 39. With a  $\delta = 0.08$  and threshold 33.67, the heavy items are 45, 42, 48, 73, 47, 55, 53, 39. The uncertainty curves have been chosen from  $10!$  sequences and shown in Figure 8.

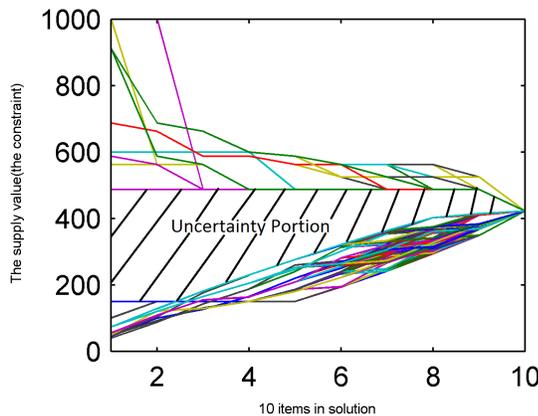


Figure 8. Uncertainty Portion with 10 items and  $\delta = 0.08$

Because the heavy part contains items that are close to the threshold, the uncertainty portion shows a similar shape as in Figure 4, Figure 5 and Figure 6.

## VI. CONCLUSION AND FUTURE WORK

The experimental evidence in the previous section shows that the uncertainty of the constraint in the 0-1 knapsack problem as the items are showing one by one is related with the threshold. As the heavy items are easier to identify given the smaller  $\delta$  that determine the threshold, the uncertainty of the constraint will be much higher. The attacker won't obtain much until in the very late phase that all items are observed as shown in Figure 7. Based on this result, Load Balance algorithm in DGI will need some strategies to prevent this implicit information leakage. For example, Load Balance should pick heavier items and lighter items with large differences as the solution for 0-1 knapsack problem to reduce the information leakage.

This paper focused on quantifying the information flow in a CPS. Given the cyber algorithm used in FREEDM (a smart grid CPS), this paper presents analysis of the information

leakage as physical values are observed. It also provides a method to quantify the information flow through the advice tape for the algorithm. For the future direction, a general method to quantify the information leakage based on the advice tape should be presented and applied for any CPS.

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